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Rapport: Learning the parts of objects by non-negative matrix factorization

1. NMF Principal (non-negative matrix factorization)

NMF is a matrix decomposition method with constraints that all elements in a matrix should be non-negative. That is why NMF is distinguished from other methods.

NMF is a new matrix decomposition algorithm, which overcomes many traditional matrix decomposition problems. And it provides a deeper view of data by finding meaningful solutions in context.

NMF decompose the non-negative matrix by looking for the low rank. The basic principal of NMF can be simply described as: For any given non-negative matrix A, the NMF algorithm can find a non-negative matrix U and a non-negative matrix V(A=U*V). As a result, the non-negative matrix A is decomposed into product of the two non-negative matrices. Since all the matrices contain only non-negative elements, a column vector in the original matrix A can be interpreted as the weighted sum of all the column vectors (called the base vectors) in the left matrix U. And the weighting coefficients correspond to the column vectors in the right matrix V. This method of representation based on the base vector composition has a very intuitive semantic interpretation, which reflects the concept of "local composition as whole" in human thinking. It is pointed out that non-negative matrix decomposition is an NP problem. (A problem is assigned to the NP (non-deterministic polynomial time) class if it is solvable in polynomial time by a non-deterministic Turing machine.) The problem can be regarded as an optimization problem. As a result, we can try to solve the matrices U and V by iterative method.

NMF algorithm provides a method based on simple iteration to solve the matrices U and V, which has the characteristics of fast convergence and small storage space. It can reduce the dimension of high-dimensional data matrix so it is very suitable to process large-scale data. The use of NMF algorithm for texts, images and other large-scale is faster and more convenient compared to traditional processing algorithms. Many people are interested in the NMF algorithm quickly. And there are many examples of applying this idea to solve the practical problems.

NMF algorithm content:
INPUT: X: The matrix to be decomposed
R: the rank of B
MAXITER: times of iteration
OUTPUT: B, H: The decomposed matrices of X
MATLAB Program:

```
dim=size(X);
X=double(X);
B=10*rand(dim(1),r);
B=B./(ones(dim(1),1)*sum(B));
H=10*rand(r,dim(2));
maxiter=100;
for iter=1:maxiter
        H=H.*(B'*(X./(B*H)));
        B=B.*((X./(B*H))*H');
        B=B./(ones(dim(1),1)*sum(B));  end
```

2. NMF compared with VD and PCA in theory

Generally, there are three principle methods who aim to reduce the dimension of data, NMF (non-negative matrix factorization), PCA (principal components analysis) and VQ (vector quantization). In this part, we will compare the method of NMF with the other two methods.

- **Common points**

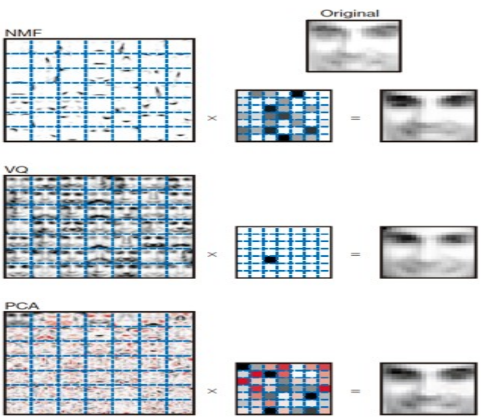
First of all, in picture processing, all three methods construct approximate factorizations of the form:

$$V_{i\mu} \approx (WH)_{i\mu} = \sum_{a=1}^{r} W_{ia} H_{a\mu}$$

When multiplying these two matrices, the dimensions of the factor matrices may be significantly lower than those of the product matrix and it is this property that forms the basis of NMF. NMF generates factors with significantly reduced dimensions compared to the original matrix. For example, if V is an m × n matrix, W is an m × p matrix, and H is a p × n matrix then p can be significantly smaller than both m and n. In this way, we can successfully reach the goal of reducing the dimension.

- **Differences**

The differences between PCA, VQ and NMF arise from different constraints imposed on the matrix factors W and H.
PCA: the columns of W should be orthonormal and the rows of H should be orthogonal to each other. So, each face is approximated by a linear combination of all the basis images.
VQ: each column of H is constrained to be a unary vector. In other words, every face (column of V) is approximated by a single basis image (column of W) in the factorization.
NMF: all the elements in W and F shouldn't be negative. As no subtractions can occur, the face can been seen as an addictive combination of multiple basis images. In this way, we can say that NMF learns a parts-based representation.



Just as shown in the picture at left:
The method NMF learns a parts based representation of Faces. VQ and PCA learn holistic representations.

3. Application and Limitation

Apart from face recognition, NMF method is also used in semantic topic analysis(Document clustering), road identification, Space Target Image Compression and Recognition and so on.

Document clustering: In order to obtain a more exact result, we start by defining a term-frequency vector of each word, total number and percentage of documents containing the word.All vector combine a matrix which we use to do NMF analyse. Using NMF, a document is a combination of basic latent semantics, making it more meaningful in the text field.

Road identification: Visual navigation is an important means of automatic vehicle navigation. It can identify the accessible area of vehicles and make it safe to complete the navigation task. Through the feature extraction algorithm, we can get the features of different regions from the images, and then use the classification algorithm to complete the recognition function.The road image feature is extracted by non-negative matrix decomposition(NMF). The experimental results show that the algorithm based on NMF is better than the traditional color and texture classification system.

Limitation:

NMF is more successful in face recognition and semantic topic analysis, but it does not mean that it can be used for any data, such as capturing images taken at non-fixed points. For this complex problem, you need multiple hidden variables structure model (similar to DL), rather than NMF that contains only one hidden variable. In addition, although non-negative constraints can be part-based representation of learning, they are also inadequate in terms of coding relevance, NMF only constrains the non-negative W and H, without taking into account the correlation between the elements of W and H.

4. Conclusion

In this course of neuroscience, we have analysed and learned the principles and characteristics of the method of non-negative matrix factorization, starting from an article of reference.This process of learning makes NMF a clear conception for us and also leads us to explore more about domain of image processing, semantic topic analysis and other domain that NMF can be applied.

5. References:

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